Stochastic Spectral Methods for Uncertainty Propagation in Numerical Models

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Summary. This talk reviews the stochastic spectral methods for the propagation of parametric uncertainties in a numerical model. The alternative non-intrusive and Galerkin methods for the definition of the spectral expansion of an uncertain model output are introduced, and associated computational strategies are discussed. Examples are shown to highlight the interest of these methods and recent developments.

1 Content

The constant development of numerical methods and computational resources allow the simulation of more and more complex systems with ever increasing accuracy. As a result, numerical simulations are today widely used, in both academia and industry, to study phenomena and systems that would be hardly, or costly, investigated by means of experimental approaches. However, modeling improvements often raise the necessity to provide a more complete and accurate information regarding the input-data of the simulation (boundary and initial conditions, geometry, external forcing, material properties, model constants, . . .). In many situations, the information needed is unfortunately subjected to uncertainty, either because of an inherent variability of the system studied or due to issues in identifying the values of the parameters involved in the model. Therefore, it is critical to assess the impact of such model input uncertainties on the numerical predictions.

Classically, the propagation of uncertainties in a numerical model is treated in a probabilistic framework, where the input-data are regarded as random quantities with prescribed probability law, leading to the problem of characterizing the random model output or solution. This can be achieved for instance by means of simulation approaches (*e.g.* Monte Carlo methods), where one samples the input to generate a sample set of output that serves subsequent analysis (moments estimation, reconstruction of probability density functions, sensitivity analysis, . . .). Such approaches are robust and present the advantage of reusing deterministic simulation tools, but they can be computationally very expensive when the resolution of the model is costly.

In this talk, I will review stochastic spectral methods [1], where the uncertain model solution is seen as a functional of the random input. Owing to the introduction of a suitable functional basis spanning the random input space, the objective is then to approximate the model output as convergent Fourier-like series. Compared to the simulation approaches, stochastic spectral methods aim at exploiting the (usually) smooth dependence of the model solution with respect to the input, in order to reduce the computational complexity (spectral convergence rate), while the functional representation greatly facilitate the analysis of the solution's variance to separate for instance the respective impact of different source of uncertainty. The determination of the series amounts to the computation of a set of deterministic coefficients representing the coordinates of the random solution in the stochastic basis. Two classes of methods can be used for the computation of these coefficients. Non-intrusive (NI) methods use a sample set of deterministic simulations to compute the coefficients, by solving a problem that depends on the selected definition of the sought approximation (projection, interpolation, least square residual or Bayesian inference). Alternative to the NI methods, the stochastic Galerkin methods uses the random model equations to reformulate a problem for the series coefficients of its solution, with possibly the need for a significant adaptation of the solvers.

Examples of applications will be shown, for linear and nonlinear models, highlighting recent advances in stochastic spectral methods (in particular reduced basis methods and stochastic adaptivity) which aim at improving computational efficiency.

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References

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