Optimal frequency sweep method in multi-rate circuit simulation

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Summary. We present a new approach for the computation of a not a-priori known, time-varying frequencies in a multi-rate circuit simulation. Typical examples are the start-up simulation of oscillators, or circuits with frequency modulation. The method is based on the optimization of the smoothness of the multi-rate solution, which is in turn essential for the efficiency of the computation.

1 Introduction

Widely separated time-scales occur in many radio-frequency (RF) circuits such as mixers, oscillators, PLLs, etc., making the analysis with standard numerical methods difficult and costly. Low frequency or baseband signals and high frequency carrier signals often occur in the same circuit, enforcing very small time-steps over a long time-period for the the numerical solution, which results in prohibitively long runtimes.

A method to circumvent this bottleneck is to reformulate the ordinary circuit DAEs as a system of partial DAEs (multi-rate PDAE). The method was first presented in [5] for the computation of steady states. The technique was adapted to the transient simulation of driven circuits with a-priori known frequencies in [7,9]. A generalization for circuits with a-priori unknown or time-varying frequencies was developed in [3, 4, 6].

Here, we present a new approach for the computation of a not a-priori known, time-varying frequency, which is driven by the requirement to have a smooth multi-rate solution, crucial for the efficiency of the computation.

2 The multi-rate circuit simulation problem

We consider circuit equations in the charge/flux oriented modified nodal analysis (MNA) formulation, which yields a mathematical model in the form of a system of differential-algebraic equations (DAEs):

$$\tfrac{d}{dt}q\big(x(t)\big) + g\big(x(t)\big) = s(t). \tag{1}$$

To separate different time scales the problem is reformulated as a multi-rate PDAE, i.e.,

$$\Big(\tfrac{\partial}{\partial\tau} + \omega(\tau)\,\tfrac{\partial}{\partial t}\Big)q\big(\hat{x}(\tau,t)\big) + g\big(\hat{x}(\tau,t)\big) = \hat{s}\big(\tau,t\big). \tag{2}$$

If the new source term is chosen, such that $s_\theta(t) = \hat{s}\big(t,\Omega_\theta(t)\big)$, where $\Omega_\theta(t) = \theta + \int_0^t \omega(s)\,ds$, then a solution $\hat{x}$ of (2) determines a family $\{x_\theta : \ \theta \in \mathbb{R}\}$ of solutions for

$$\tfrac{d}{dt}q\big(x(t)\big) + g\big(x(t)\big) = s_\theta(t), \tag{3}$$

by $x_\theta(t) = \hat{x}\big(t,\Omega_\theta(t)\big)$.

Although the formulation (2) is valid for any circuit, it offers a more efficient solution only for certain types of problems. This is the case if $\hat{x}(\tau,t)$ is periodic in t and smooth with respect to τ. In the sequel we will consider (2) with periodicity conditions in t, i.e., $\hat{x}(\tau,t) = \hat{x}(\tau,t+P)$ and suitable initial conditions $\hat{x}(0,t) = X_0(t)$. Here P is an arbitrary but fixed period length.

3 Meaning and suitable choice of $\omega(\tau)$

Note that $\omega(\tau)$ can, with a corresponding choice of $\hat{s}(\tau,t)$, be chosen arbitrarily. This freedom may be used to facilitate an efficient numerical solution of (2). The smoothness of $\hat{x}(\tau,t)$ is essential for the efficiency of classical solvers. Therefore, we require

$$\int_{\tau_1}^{\tau_2}\int_0^P \Big|\tfrac{\partial}{\partial\tau}\hat{x}(\tau,t)\Big|^2\,dt\,d\tau \to \min. \tag{4}$$

in order to determine $\omega(\tau)$. For frequency modulated oscillations one obtains indeed the instantaneous frequency as $\frac{\omega(t)}{P}$, while $\hat{x}(\tau,t)$ is constant with respect to τ. For some examples a (nearly) optimal $\omega(t)$ might be known in advance, while in other case (e.g. start-up of an oscillator) it might be necessary to determine $\omega(t)$ during the simulation, by enforcing the smoothness condition (4).

4 Discretization

We discretize (2) with respect to τ by a Rothe method using a linear multi step method (e.g. Gear's BDF or the trapezoidal rule). This results in a periodic boundary value problem in t of the form

$$\omega_k \tfrac{d}{dt} q_k\big(X_k(t)\big) + f_k(X_k,t) = 0, \qquad (5)$$
$$X_k(t) = X_k(t+P),$$

where $X_k(t)$ is the approximation of $\hat{x}(\tau_k,t)$ for the k-th time step τ_k, while ω_k is an approximation of $\omega(\tau_k)$. An optimal ω_k is determined with the condition

$$\int_0^P \big|X_k(t) - X_{k-1}(t)\big|^2 dt \to \min, \qquad (6)$$

which is a good approximation of condition (4).

The periodic problem (5) can be solved by a collocation or Galerkin method, where $X_k(t)$ is expanded in a periodic basis $\{\phi_k\}$ (as a Fourier, B-spline, or wavelet basis) and tested at collocation points or integrated against test functions, respectively. This leads to a nonlinear system of equations for the coefficients $c_{k,\ell}$ of the basis expansion $X_k(t) = \sum_\ell c_{k,\ell}\,\phi_\ell(t)$. Here, the condition (6) is replaced by the condition

$$\sum_\ell \|c_{k,\ell} - c_{k-1,\ell}\|_2^2 \to \min. \qquad (7)$$

5 Example: A Phase Locked Loop

The method has been tested our method on several circuits. To solve the periodic problem we have used an adaptive spline wavelet method described in [1,2]. We show results from the multi-rate simulation of a Phase Locked Loop (PLL) with a frequency modulated input signal. Both the frequency parameter $\omega(\tau)$ (Fig. 1) determined by our method, and the control signal of the oscillator (Fig. 2) reflect perfectly the instantaneous frequency. The feedback signal in Fig. 3 shows that the computed $\omega(\tau)$ leads indeed to optimal smoothness.

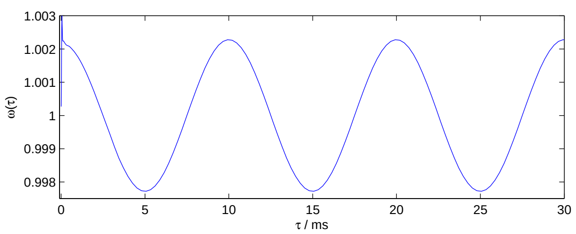

Fig. 1. Plot of simulated $\omega(\tau)$.

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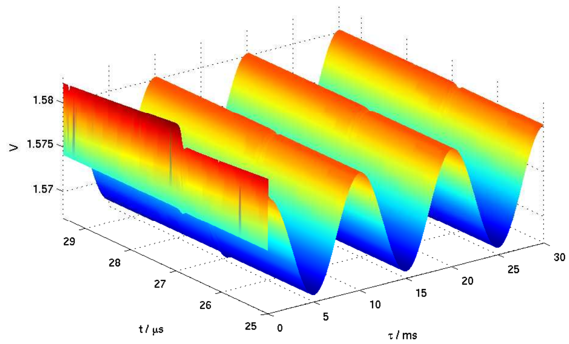

Fig. 2. Control signal for the oscillator in the PLL (multi-rate).

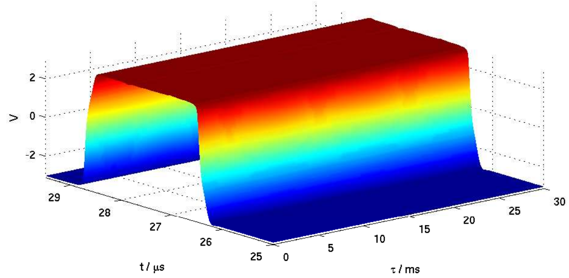

Fig. 3. Feedback signal of the PLL (multi-rate).

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