Polynomial Fitting of Nonlinear Sources with Correlating Inputs

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Summary. This paper proposes methods to improve the LSE polynomial fitting of bivariate nonlinear VCCS sources for distortion contribution analysis. The main problem in fitting is usually the fact that the input signals correlate strongly. It is shown that the correlation can be reduced by perturbing the input signals, which highly improves the quality of the fit. The ways to recognize a bad fit are discussed and the comparison between general, Chebychev and perturbed polynomial is performed.

1 Introduction

Volterra analysis is a powerful tool for finding the contributions of nonlinear distortion [1],[2]. However, it relies on the use of polynomial models that are usually not available. Volterra models are mostly built using high-order derivatives of the I-V and Q-V functions, but a polynomial model fitted using existing DC or AC I-V data is certain to converge over the data range. However, the fitting suffers from ill-conditioning resulting in non-physical coefficients. This paper illustrates heuristic methods how to recognise a bad fit, and ways to prevent it. We will propose a method that guarantees a physically meaningful fitted polynomial models are that converge over a the required signal range.

Here we utilize the frequency domain polynomial fitting [3] using large-signal voltage and current spectra of each nonlinear VCCS obtained from HB simulation. Hence, we can monitor the quality of the fitting of each VCCS individually and improve the fitting of those sources that suffer from a bad fit. The I_{DS}-V_{GS}-V_{DS} current source of a MOS transistor [4] is used as an example. Its frequency domain polynomial can be written as

$$\begin{aligned} I_{DS}(f) &= K_{00} + K_{10}\cdot V_{10}(f) + K_{20}\cdot V_{20}(f) \\ &+ K_{30}\cdot V_{30}(f) + K_{40}\cdot V_{40}(f) + K_{50}\cdot V_{50}(f) \\ &+ K_{01}\cdot V_{01}(f) + K_{02}\cdot V_{02}(f) + K_{03}\cdot V_{03}(f) \\ &+ K_{11}\cdot V_{11}(f) + K_{21}\cdot V_{21}(f) + K_{12}\cdot V_{12}(f) \end{aligned} \quad , (1)$$

where $V_{ij}(f)$ means the spectrum of a product term $v_{GS}{}^{i}v_{DS}{}^{j}$ - e.g. $V_{30}(f)$ are obtained by convolving $V_{10}(f)$ twice with itself. Controlling voltages V_{GS} and V_{DS} often correlate rather strongly, and this easily causes excessively strong V_{DS}-related product terms in the polynomial on lines 3 and 4 in (1).

2 Ways to Recognize a Bad Fit

The LSE fitting [5] tries to match the output current spectrum as accurately as possible, which is relatively easy to achieve. However, the model may still show excessive curvature outside the data range and does not make sense physically. In order to monitor the quality of the fit, we can calculate the estimated variance for fitted parameters [5]. This easily calculated numerical measure shows if the result is uncertain and has a risk. The condition number cond calculated from the singular values of the model matrix M in $y_{est} = Mc$, $c = (M'M)^{-1}(M'y)$ gives similar indication [5]. If cond(M) is high, the model functions in M most probably correlate. Visually the quality of the fit can be illustrated by the model's capability to imitate the I-V shape of the original transistor. In Fig. 1 the I-V-curves of the original model are plotted on top of a narrow I_{DS}-V_{DS} swing (black) caused by the strongly correlating V_{GS} and V_{DS}. The general frequency domain polynomial (thick line) that is fitted using the narrow data range show excessive curvature outside the data trajectory, which implies that the model - though accurate - is non-physical.

3 Methods to Improve the Fitting

Several approaches have been proposed to guarantee that the fitted polynomial is physically meaningful. One is to reduce the order of the model (especially of V_{DS}-related terms). This helps, but also limits the usability of the model in highly nonlinear applications. Another method is to apply orthogonal series expansion like Chebychev series to reduce the correlation between odd and even degree terms. This works rather well with a multitone spectra, too: if the DC content is eliminated in the signals to be multiplied, the original frequency components will be attenuated in the resulting higher-order spectrum. The effect of this is shown in Fig. 2, in which the increase of cond(M) between different order spectra $V_{ij}(f)$ is shown. It can be seen that condition number of the terms in Chebychev polynomial is indeed lower up to V_{50} but then increases above the general polynomial. In fact, a Chebychev polynomial can not break the correla-

Table 1: Coefficients and the reliability of general (C_1), Chebychev (C_2) and perturbed (C_3) polynomial.

Terms	DC	V_{10}	V_{20}	V_{30}	V_{40}	V_{50}	V_{01}	V_{02}	V_{03}	V_{11}	V_{21}	V_{12}
C_1	0.33	1.29	1.78	0.35	-0.98	-0.25	2.3m	160u	20u	10.8m	8.9m	290u
C_1/sigma1	6310	2080	166	13	155	77	33	1.1	0.4	4.3	0.9	0.2
C_2	0.870	1.32	0.80	72.6m	-0.12	-15m	2.5m	80u	10u	10.8m	8.8m	290u
C_2/Sigma2	319	380	148	10.5	154	77	4.5	1.1	0.4	4.3	0.9	0.2
C_3	0.33	1.29	1.78	0.39	-0.99	-0.24	2.3m	140u	10u	10.5m	17m	640u
C_3/Sigma3	7640	3650	1300	97	174	140	60	18.6	2.8	82.4	17.4	6.9

tion between V_{GS} and V_{DS}, which is the main problem in the 2-D fitting. By fitting the V_{GS} and V_{DS} related polynomial terms in two phases (V_{GS} first, then V_{DS}), helps a little. One could also use the partial derivatives of I_{DS} to aid the fitting, as is often done in fitting DC models. However, that does not break the V_{GS}-V_{DS} correlation, either.

The obvious solution in breaking V_{GS}-V_{DS} correlation is to perturb either one of them [6]. This means that the data used for fitting is different from the one used for distortion contribution analysis, and we need to maintain the same peak amplitudes of the control signals. Also, two simulations are needed: one for fitting the models, and one for the real signals for the contribution analysis. This is implemented so that only the hard-to-fit I-V or Q-V sources are recognized, and fitted in a separate HB simulation loop where only that specific source is simulated and refitted. It can be seen in Fig. 2 that when a half the amplitude of the f_1 tone (900 mV$\angle 45^o$) is added to the f_2-f_1 of V_{DS}, the cond(M) (especially cross-products) are highly reduced.

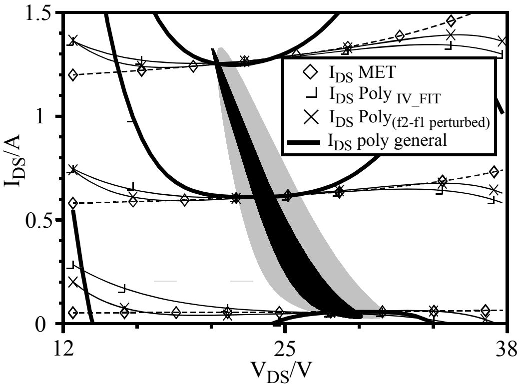

Fig. 1 I_{DS} - V_{DS} voltage swings with IV curves.

The IV-curves of this polynomial can be seen in Fig. 1 (X marker) and it behaves surprisingly well. The perturbation causes now wider data trajectory (grey) indicating lower correlation between V_{GS} and V_{DS}. With less correlating data it is possible to fit a frequency domain polynomial that is accurate and able to imitate the shape of the actual IV-curves. In fact, this polynomial is more accurate than the IV-fitted polynomial(⌟ marker) based on the actual DC curves.

To observe the results further the Table 1 shows the coefficients of three different frequency domain polynomials and their reliability. The higher the reliability figure the smaller the deviation. It can be seen that the perturbation definitely increases the reliability of all terms, especially in the higher order terms and cross products.

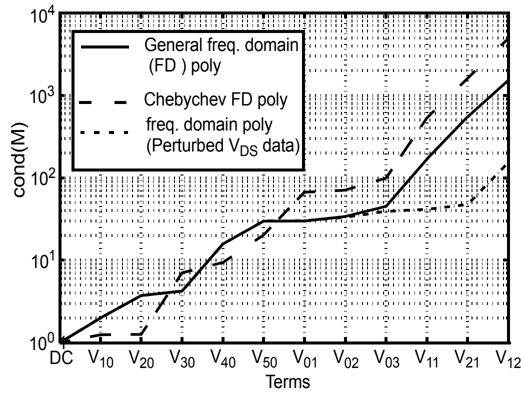

Fig. 2 Increase of the cond(M) term by term.

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