Streamer Inception and Propagation from Electric Field Simulations

Thomas Christen

ABB Switzerland Ltd., Corporate Research, CH-5405 Baden-Dättwil, Switzerland

Summary. A recipe is introduced for the determination of streamer inception regions and streamer propagation paths from the electric background field. The method is based on the equivalence of the streamer inception integral with a first order partial differential equation (PDE). It can be easily used in modern commercial multi-physics simulation tools, and circumvents the cumbersome search for critical field lines and their postprocessing.

Introduction

Streamer inception (SI) at an electrode and subsequent streamer propagation (SP) towards the counter electrode are initial steps of dielectric gas breakdown in nonuniform high electric fields [1]. Often, the aim of electric field calculations is to identify the locations where SI can occur and to determine how far streamers can propagate. This note introduces a simple procedure to calculate SI and SP from quasi-static electric background fields using common SI and SP criteria [1,2].

We thus assume that the solution of the Laplace equation for the electric potential $U(\mathbf{x})$ in the compact spatial region of interest, $\Omega \subset \mathbf{R}^3$, is given for appropriate boundary conditions. The boundary of Ω is denoted by $\partial\Omega$. Let the potential be positive at the electrode under consideration, $\partial\Omega_0 \subset \partial\Omega$, i.e., $U_0 = U(\mathbf{x}) > 0$ for $\mathbf{x} \in \partial\Omega_0$ (Dirichlet boundary condition). Assume further that the potential at the counter electrode(s) is smaller, for instance grounded, such that the field lines of the electric field, $\mathbf{E} = -\nabla U$, point away from $\partial\Omega_0$. The *SI criterion* is associated with the critical electron avalanche size and is formulated as an integral condition to the effective ionization coefficient $\alpha(E)$ along a field-line path γ where α is positive and which ends at $\partial\Omega_0$ [1,2],[1]

$$\int_\gamma \alpha(E)\, ds \geq C_{\text{crit}} \tag{1}$$

with field strength $E(\mathbf{x}) = |\,\mathbf{E}\,|$. For a field distribution $\mathbf{E}(\mathbf{x})$ in an arbitrary geometry, it is not a priori obvious which are the critical field lines satisfying Eq. (1); they are not necessarily related to electrode locations with maximum field.

[1] The integral (1) gives $\ln N/N_0$, where N is the number of electrons in an avalanche, and N_0 is the number of starting electrons. For negative α, electrons recombine or are attached.

The required search for and extraction of information on field lines from electric simulations for realistic geometries, as it is needed for (1), is usually not a feature provided by typical commercial E-field simulation tools. But we will show that there is a simple way to determine the critical SI region $\Gamma \subset \Omega$, and thus the critical electrode region, $\partial\Gamma_0 = \Gamma \cap \partial\Omega_0$, without cumbersome postprocessing.

Streamer Inception (SI)

We introduce the scalar field variable $\phi(\mathbf{x})$, which satisfies the 1$^{\text{st}}$ order PDE

$$-\mathbf{v}\cdot\nabla\phi = \alpha(E)\Theta(\alpha) \tag{2}$$

where Θ is the Heaviside theta-function such that the right side vanishes for negative α, and

$$\mathbf{v}(\mathbf{x}) = \frac{\mathbf{E}}{E} \tag{3}$$

is the normalized vector field along the field lines. Equation (2) means that the derivative of ϕ along the backward direction of the field lines (i.e., towards the electrode $\partial\Omega_0$), equals α. Hence the solution of Eq. (2) is the integral of α along field lines and equal to the streamer integral (1), provided $\phi = 0$ in regions where $\alpha \leq 0$. The latter condition is ensured by using a homogeneous Dirichlet boundary condition, $\phi = 0$, at the counter electrode(s), where the flow lines of $\mathbf{v}$ end. The theta-function in Eq. (2) ensures integration only for $\alpha \geq 0$. The SI region Γ, where streamers will emerge, is then obtained from $\phi(\mathbf{x}) \geq C_{\text{crit}}$. Note that because $\Gamma \subset \Omega$ is a volume region, the procedure allows also the determination of electrodeless SI.

Steamer Propagation (SP)

A SP model has to predict where and how far the emerging streamers will go. If they reach the counter electrode, dielectric breakdown may occur. Streamer-to-leader transition is not discussed here [4]. A simple SP model makes use of the observation that a streamer length increase requires a roughly constant voltage drop, which can be associated with a field E_{s} along the streamer path. The potential drop along a streamer of length s is then [4]

$$U_\mathrm{s}(s) = U_\mathrm{s,0} + E_\mathrm{s} s \tag{4}$$

where $U_\mathrm{s,0}$ can be interpreted as the streamer head voltage. If it is assumed that streamers follow field lines, the path can be found by solving the ordinary differential equation (ODE) for the location $\mathbf{x}(t)$ of, say, the streamer head

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x})\, h(\Delta U, t) \tag{5}$$

with initial condition $\mathbf{x}(0) \in \partial\Gamma$ (or, here, Γ_0) for $t = 0$, and $\Delta U = U_0 - U(\mathbf{x})$ is the voltage drop along the streamer line. Note that t is equal to the streamer length s because $\mid \mathbf{v} \mid = 1$ ($\mathbf{v}$ is not the true streamer velocity but its direction vector; the true speed, which is typically of the order of mm/ns [3] is not needed for determining the streamer length for many practical cases). The prefactor $h(\Delta U, t)$ is either 1 or 0, depending on whether the SP criteria is satisfied or not. The prefactor h ensures that the streamer stops if the local potential drop is insufficient for further propagation. For brevity, the considerations are here restricted to $U_\mathrm{s,0} = 0$, where $h = \Theta(\Delta U - E_\mathrm{s} s)$.

The assumption that streamers follow field lines may not always be valid, as was critically discussed in Refs. [4, 5]. Nevertheless, generalized models might be taken into account in our simple propagation model by a redefinition of $\mathbf{v}(\mathbf{x})$ in Eq. (5) [4].

Results

The incorporation of our SI approach in typical commercial multi-physics simulation tools, which usually solve 2$^{\mathrm{nd}}$ order PDEs, requires a mimicry of the 1$^{\mathrm{st}}$ order PDE (2) with a 2$^{\mathrm{nd}}$ order PDE of the form $D\Delta\phi - \mathbf{v}\cdot\nabla\phi = \alpha(E)$. The structural difference between them leads to a singularly perturbed problem (i.e., the limit $D \to 0$ is not equivalent to $D = 0$). However, the solution of Eq. (2) can be approximated with sufficient accuracy for practical purposes, if D is small enough and the boundary conditions to ϕ are appropriately chosen. In particular, the disturbance of the solution by the boundary condition at $\partial\Omega_0$ should be negligibly small. Because for $D = 0$ one has $\partial_s\phi = \alpha$ at $\partial\Omega_0$, one must have $D \ll 1/\mid \partial_E \ln(\alpha)\partial_s E \mid$, and the boundary condition must be $\mathbf{n}\cdot\nabla\phi = -\alpha$, where $\mathbf{n}$ denotes the surface normal vector at $\partial\Omega_0$.

As an example, we consider a tip-plate geometry in normal air, where [6] $\alpha = p[k(\frac{E}{p} - \Lambda)^2 - A]$ with $k = 1.6$ mm bar/kV^2, $\Lambda = 2.2$ kV/(mm bar), $A = 0.3$ 1/(mm bar), p = 1 bar, tip-plate distance 19 cm, tip radius 1 cm, and $E_\mathrm{s} = 0.5$ kV/mm. A result for $U_0 = 80$ kV is shown in Fig. 1; the SI voltage, when the first streamer appears is ca. 67 kV. The SI region is visible as the small dark area in front of the tip. The voltage when the first streamer crosses the gap is for this case $U_\mathrm{bd} = 95$ kV.

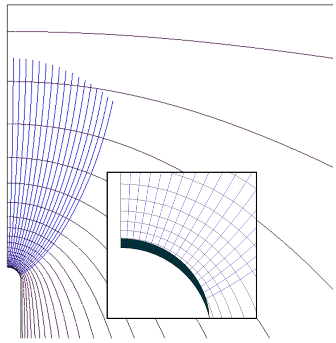

Fig. 1. Tip-plate geometry with simulated equipotential curves, SI region in front of the tip (insert), and streamer lines (simulation tool: Comsol; streamer lines with "particle tracing" feature).

Conclusion

Streamer lines associated with the common SI and SP criteria used in electrical engineering, can be calculated directly from standard multi-physics simulation tools without cumbersome postprocessing of electric field line data, provided the tool exhibits at solvers for an additional linear PDE (for SI) and an ODE (for SP).

References

1. Boeck W. and Pfeiffer W., *Conduction and Breakdown in Gases*, in "Wiley encyclopedia of electrical and electronics engineering", Vol 4, J. G. Webster (editor), J. Wiley & Sons Inc., New York, 1999, pp. 123-172.
2. Gallimberti I., *The mechanism of the long spark formation*, Colloque de physique 40, 1979, pp. 193-250.
3. T. Briels et al. *Positive and negative streamers in ambient air: measuring diameter, velocity and dissipated energy*, J. Phys. D: Appl. Phys 41, 234004 (2008).
4. T. Christen et al, *Streamer line modeling*, Scientific computing in electrical engineering SCEE 2010, Eds. B Michielsen and J-R. Poirier, Springer (2012), pp. 173.
5. A. Pedersen et al, *Streamer inception and propagation models for designing air insulated power devices*, CEIDP Conference Material, Virginia Beach (Oct. 2009).
6. K. Petcharaks, *Applicability of the streamer breakdown criterion to inhomogeneous gas gaps*, ETH Thesis No. 11192 (1995).