Reduced Basis Modeling for Time-Harmonic Maxwell's Equations

Peter Benner and Martin Hess

Max Planck Institute for Dynamics of Complex Technical Systems, Sandtorstr. 1, 39106 Magdeburg, Germany
benner@mpi-magdeburg.mpg.de, hessm@mpi-magdeburg.mpg.de

Summary. The Reduced Basis Method generates low-order models of parametrized PDEs to allow for efficient evaluation of parametrized models in many-query and real-time contexts.

We show the theoretical framework in which the Reduced Basis Method is applied to Maxwell's equations and present first numerical results for model reduction in frequency domain.

1 Introduction

The Reduced Basis Method (RBM) generates low order models for the efficient solution of parametrized PDEs in real-time and many-query scenarios. The RBM employs rigorous error estimators to perform the model reduction and measure the quality of the reduced simulation. In recent years, the RBM has been developed to apply to a wide range of problems, of which [1] and the references therein, give an overview.

We address the use of the RBM in time-harmonic electromagnetic problems, which can exhibit parameter variations in geometry, material coefficients and frequency. We use the RBM in large 3D problems, that arise in the analysis of microscale semiconductor structures.

2 Model Problem

As an example model, we consider the coplanar waveguide, depicted in Fig. 1. The model setup is contained in a shielded box with perfect electric conducting (PEC) boundary. We consider three perfectly conducting striplines as shown in the geometry. The system is excited at a discrete port and the output is taken at a discrete port on the opposite end of the middle stripline. These discrete ports are used to model input and output currents/voltages.

2.1 Constitutive Equations

We consider the second order time-harmonic formulation of Maxwell's equations in the electric field E

$$\nabla \times \mu^{-1} \nabla \times E + i\omega\sigma E - \omega^2 \varepsilon E = i\omega J \quad \text{in } \Omega, \quad (1)$$

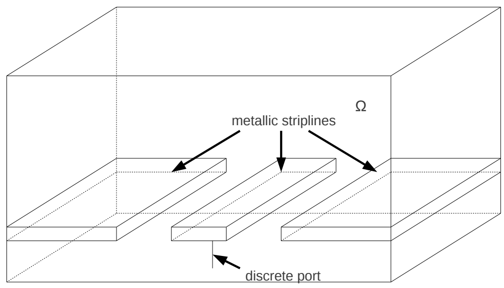

Fig. 1. Geometry of coplanar waveguide.

subject to zero boundary conditions

$$E \times n = 0 \quad \text{on } \Gamma_{\text{PEC}}. \quad (2)$$

We use the weak formulation to (1) with bilinear form $a(\cdot,\cdot;\nu)$ and linear form $f(\cdot;\nu)$ as

$$a(E(\nu), v; \nu) = f(v; \nu) \quad \forall v \in X, \quad (3)$$

where $\nu \in \mathscr{D} \subset \mathbb{R}^p$ denotes the parameter vector, $E(\nu)$ is the parameter-dependent electric field, v a test function and X the $H(\text{curl})$-conforming finite element space, discretized with Nédélec finite elements.

All the model problems which are used in this work have been developed in the MoreSim4Nano project [3].

3 Reduced Basis Method for time-harmonic EM-problems

The aim of the RBM is to determine a low order space X_N of dimension N, which approximates the parametric manifold

$$M^\nu = \{E(\nu) | \nu \in \mathscr{D}\} \quad (4)$$

well. Given such a space X_N, it is possible to gain accurate approximations $E_N(\nu)$ to $E(\nu)$ by solving (3) in X_N

$$a(E_N(\nu), v_N; \nu) = f(v_N; \nu) \quad \forall v_N \in X_N, \quad (5)$$

i.e. projecting (3) onto X_N.

An integral part in the model reduction are error estimators $\Delta_N(\nu)$, which give rigorous bounds to the approximation error in the $H(\text{curl})$ norm

$$\|E(\nu) - E_N(\nu)\|_X \leq \Delta_N(\nu). \tag{6}$$

Additionally, the RBM requires to have fast evaluations of the error estimator in the sense that the complexity is $O(N)$, i.e. independent of the large discretisation of the full model. The necessary requirement is an affine decomposition of the forms as

$$a(E(\nu), v; \nu) = \sum_{q=1}^{Q} \Theta^q(\nu) a^q(E(\nu), v). \tag{7}$$

3.1 Error Estimation

The error estimator in the field is given by

$$\Delta_N(\nu) = \frac{\|r(\cdot;\nu)\|_{X'}}{\beta_{LB}(\nu)}, \tag{8}$$

with $\|r(\cdot;\nu)\|_{X'}$ the dual norm of the residual and $\beta_{LB}(\nu)$ a lower bound to the inf-sup stability constant.

For error estimation in the output, the adjoint equation is solved to obtain the dual residual $r^{du}(\cdot;\nu)$, such that

$$\Delta_N^s(\nu) = \frac{\|r^{pr}(\cdot;\nu)\|_{X'}}{(\beta_{LB}(\nu))^{1/2}} \frac{\|r^{du}(\cdot;\nu)\|_{X'}}{(\beta_{LB}(\nu))^{1/2}}, \tag{9}$$

gives rigorous bounds in the output. Here, $r^{pr}(\cdot;\nu)$ denotes the original, primal residuum.

3.2 Geometric Parameters

To consider the linear combination of snapshots for different geometries, the PDE is transformed from the parameter-dependent domain $\Omega(\nu)$ to a parameter-independent reference domain $\Omega(\overline{\nu})$.

Given a domain decomposition of $\Omega(\overline{\nu})$, such that each domain under consideration can be found under affine transformations of the subdomains, the affine decomposition (7) is possible and therefore allows the Reduced Basis model reduction.

4 First Numerical Results

The full simulation has been performed with the finite element package FEniCS using a discretization with first order Nédélec finite elements. For our first numerical experiments, we used a coarse discretization of 2048 degrees of freedom. To work with geometric variations, a larger resolution is required.

Fig. 2 shows the transfer function of the coplanar waveguide. In our simulations, we applied the RBM over the frequency range $[0.6, 3.0]$ GHz. The time-harmonic equations are already stated in the affine form (7).

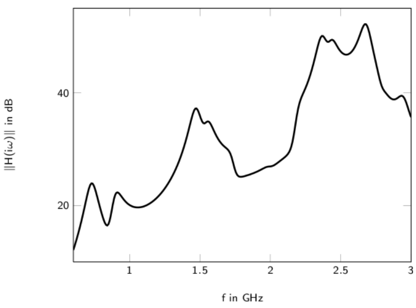

Fig. 2. Frequency response of coplanar waveguide.

In Fig. 3, the relative approximation error for the order $N = 30$ and $N = 50$ are shown. In the case of $N = 50$, the relative error is already below 0.01%. Overall, the RBM achieves fast convergence in that the full model is approximated to machine precision with a space X_N of order 75 for the considered parameter range.

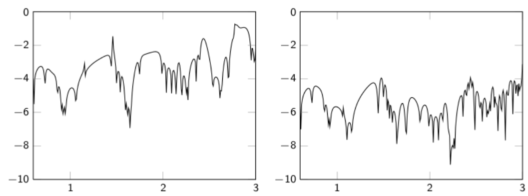

Fig. 3. Log-plot of relative error for $N = 30$ (left) and $N = 50$ (right).

References

1. G. Rozza, D.B.P. Huynh and A.T. Patera,
   Reduced Basis Approximation and a Posteriori Error Estimation for Affinely Parametrized Elliptic Coercive Partial Differential Equations,
   Arch. Comput. Methods Eng. (2008) 15:229-275.
2. R. Hiptmair,
   Finite Elements in computational electromagnetism,
   Acta Numerica (2002) 237 - 339.
3. MoreSim4Nano,
   Model reduction for fast simulation of new semiconductor structures for nanotechnology and microsystems technology,
   www.moresim4nano.org