Improved Electromagnetic Modelling and Simulation of Axial Flux Machines

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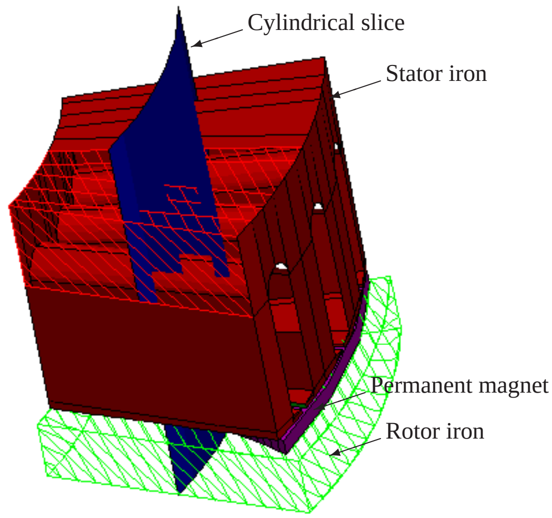
Summary. Axial-field permanent magnet synchronous machines can be tackled by means of the so-called quasi 3D approach, where the 3D problem is reduced to a family of decoupled 2D problems. This approach is placed in proper mathematical context and the modelling error is discussed. The resulting 2D problems are cast into standard radial flux topology.

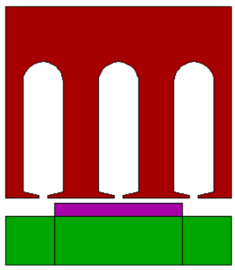
1 Introduction

Axial-field permanent magnet synchronous machines enjoy increasing importance. In particular, their flat and compact shape renders them attractive for several applications, for instance in electric vehicles, elevator drives, or wind generators. However, when it comes to electromagnetic modelling and simulation, there is much less literature and tools available compared to the standard radial flux topology. Since full transient 3D Finite Element simulations are still at the feasibility limit, the so called *quasi 3D approach* is often reported in literature, both for numerical [3] and analytical [1, 2, 4, 5] modelling.

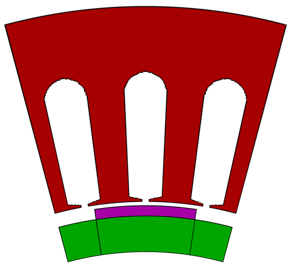
The machine geometry is represented by a number of cylindrical slices, compare Fig. 1(a). Each slice is then unrolled, which yields the flat geometry depicted in Fig. 1(b), which we will call *translational model*. Eventually, the slice might be distorted into the segment shown in Fig. 1(c), which we will call *rotational model*. The latter corresponds with the symmetry cell in the cross section of a standard radial flux permanent magnet synchronous machine, and can therefore be computed by well-established methods.



(a) 3D model of a symmetry cell cut by a cylindrical slice.



(b) Unrolling the slice yields the 2D *translational model*.



(c) The 2D *rotational model* is obtained by distortion.

Fig. 1. Geometry of an axial flux machine. Stator coils are omitted. Images (a) and (b) are taken from [4, Fig. 1], reprint with kind permission.

2 Mathematical Modelling

In references [1] – [5] it is usually taken for granted that the magnetic flux in the machine has no radial component. It is then claimed that each translational or rotational model can be analyzed separately, based on a single component magnetic vector potential. The torque of the machine is obtained by adding up the contributions of the individual slices. We will put this approach into proper mathematical context. For the purpose of this paper we restrict ourselves to a magnetostatic model for each time step.

The magnetostatic field in the 3D model is governed by $\mathrm{curl}\,\mu^{-1}\mathrm{curl}\,\mathbf{A} = \mathbf{J}$, where μ is the magnetic permeability, in general dependent on the field, $\mathbf{A}$ is the magnetic vector potential, $\mathbf{B} = \mathrm{curl}\,\mathbf{A}$, and $\mathbf{J}$ is the total current density, where $\mathrm{div}\,\mathbf{J} = 0$ holds. $\mathbf{J}$ takes into account both the stator currents as well as the permanent magnets, in terms of magnetization currents. We introduce cylindrical coordinates (r, φ, z), compare Fig. 3, left. The vector potential can be gauged such that $A_z = 0$ holds, without loss of generality. We introduce the second order differential operators

$$\Delta_{\varphi z} = \partial_\varphi \mu^{-1} \frac{1}{r^2} \partial_\varphi + \partial_z \mu^{-1} \partial_z,$$
$$\Delta_{rz} = \partial_r \mu^{-1} \frac{1}{r} \partial_r r \ + \partial_z \mu^{-1} \partial_z.$$

In the chosen gauge, the double curl equation reads

$$\begin{bmatrix} \Delta_{\varphi z} & -\partial_\varphi \mu^{-1} \frac{1}{r^2} \partial_r r \\ -\partial_r \mu^{-1} \frac{1}{r} \partial_\varphi & \Delta_{rz} \end{bmatrix} \begin{bmatrix} A_r \\ A_\varphi \end{bmatrix} = - \begin{bmatrix} J_r \\ J_\varphi \end{bmatrix}. \quad (1)$$

This system can be interpreted as a family of problems defined on cylinders $r =$const. in terms of a radial potential A_r, plus a family of problems defined on half-planes $\varphi =$const. in terms of an azimuthal potential A_φ. Both families are coupled via off-diagonal terms. It can be shown that if and only if $B_r = 0$ holds, the field can be described in terms of A_r alone. In practice, in axial flux machines $B_r \approx 0$ holds, so the first family dominates over the second.

This motivates working with the modelling assumption $B_r = 0$, that is, $A_\varphi = 0$. In this case, the first equation of (1) reduces to $\Delta_{\varphi z} A_r = -J_r$, which can be solved on each cylinder $r =$const. separately. The second equation gives rise to a residual

$$\mathscr{R} = \partial_r \mu^{-1} \frac{1}{r} \partial_\varphi A_r - J_\varphi = -\partial_r H_z - J_\varphi,$$

where $\mathbf{H} = \mu^{-1}\mathbf{B}$ holds. The residual gives an indication for the error introduced by the modelling assumption $B_r = 0$. For an interpretation see Fig. 2.

$$\int_S \mathscr{R} \, \mathrm{d}a = \int_{\partial S} \mathbf{H} \cdot \mathrm{d}\mathbf{l} - \int_S \mathbf{J}_\varphi \cdot \mathrm{d}\mathbf{a}$$

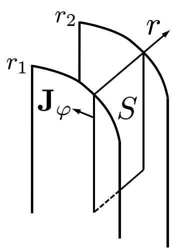


Fig. 2. The residual may be integrated over rectangle S. A non-zero residual indicates failure to fulfill Ampère's law in zr-planes.

For fixed radius r, introduce a new coordinate $\ell = r\varphi$. In coordinates (ℓ, z) the equation to be solved for A_r reads $(\partial_\ell \mu^{-1} \partial_\ell + \partial_z \mu^{-1} \partial_z) A_r = -J_r$, the governing equation for the translational model, Fig. 1(b).

From a practical point of view, existing software for the numerical analysis of standard radial flux machines should be employed. To that end, a transformation is required that maps lines $\ell =$const. to radial half-lines, while lines $z =$const. shall be mapped to concentric circles. We pick the conformal map $\mathscr{Z} = c\exp(\mathscr{W}/r) : \mathscr{W} = \ell + iz \mapsto \mathscr{Z} = x + iy = \rho \exp(i\varphi)$, for fixed $c, r \in \mathbb{R}$. The transformation is depicted in Fig. 3.

Let $A_z(x,y) = A_r(\ell, z)$, $J_z(x,y) = J_r(\ell, z)$, which yields $(\partial_x \mu^{-1} \partial_x + \partial_y \mu^{-1} \partial_y) A_z = -h^2 J_z$, with the conformal factor $h = r/\rho$. This is the usual magnetostatic 2D vector potential formulation, the governing equation of the rotational model, Fig. 1(c). The Laplace operator is invariant under conformal transformation, while the current density has to be scaled by h^2. The rotational model can therefore be solved efficiently with commercially available software, for each cylindrical slice $r =$const. that is contained in the discretization. Relevant postprocessing quantities like the torque of the machine can be computed once the solutions for the slices are available.

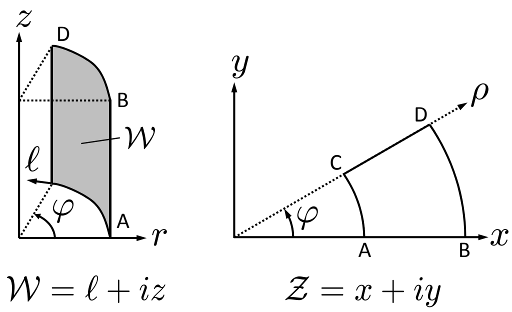


Fig. 3. The conformal map $\mathscr{Z} = c\exp(\mathscr{W}/r)$ relates the rotational to the translational model, for fixed $c, r \in \mathbb{R}$.

An example for this approach will be given in the full paper.

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