Validation of the Potential Method; comparing measurements of a dihedral with calculations

Magnus Herberthson[1,2]

[1] Sensor and EW Systems, FOI, Swedish Defence Research Agency, magnus.herberthson@foi.se,
[2] Department of Mathematics, Linköping University, magnus.herberthson@liu.se

Summary. Earlier reported is the potential method, which addresses the EFIE (Electric Field Integral Equation) or MFIE/CFIE by applying the Hodge decomposition theorem to a one-form related to the physical current **J**. In this approach, one solves for two unknown scalar potentials, Φ and Ψ, which carries the same information as **J**. Here we compare calculations on a 100° dihedral with measurements. The calculations are made on meshes with different triangle sizes, which give a simple convergence study. The computational burden is also compared with other methods.

1 Introduction

We look at the electromagnetic scattering problem in frequency domain. More precisely, we first address the Electric Field Integral Equation, EFIE, [1]. In the standard formulation using the method of moments (MoM), objects which are large compared to the wavelength will produce linear systems which easily becomes to large to solve with direct solvers. This problem can be tackled in various way, and one option is to use the potential method, [2].

This method has been reported earlier, [3], [4], demonstrating proof of concept under various circumstances. In this work, we will make more quantitative evaluations, comparing measurements on a 100° dihedral (see Fig. 1) with calculations using the potential method. We will make a simple convergence study, i.e, compare calculations using different meshes, and also compare the number of unknowns with other approaches.

2 Formulation

In standard notation, with a plane wave illuminating a PEC surface S and in an adapted ON-basis, the EFIE (electric field integral equation) reads

$$\forall \mathbf{r} \in S : -E_0 e^{-\mathrm{i}kz}\hat{\mathbf{x}} \doteq \mathrm{i}kc\mu_0(\mathbf{I} + \frac{1}{k^2}\nabla\nabla\cdot)\int_S g(\mathbf{r},\mathbf{r}')\mathbf{J}(\mathbf{r}')\mathrm{d}S'. \tag{1}$$

Here **J** is the unknown current, g is the standard Green's function and $\doteq$ means tangential equality. By the replacement

$$h(\mathbf{r},\mathbf{r}') = g(\mathbf{r},\mathbf{r}')e^{\mathrm{i}k(z-z')}, \quad \mathbf{K}(\mathbf{r}') = e^{\mathrm{i}kz'}\mathbf{J}(\mathbf{r}') \tag{2}$$

and by the application of Hodge decomposition to **K** (under the assumption that S is homeomorphic to a sphere), so that, in vector calculus notation,

$$\mathbf{K} = \nabla_S \Phi + \widehat{\mathbf{n}} \times \nabla_S \Psi,$$

we can express the EFIE in terms of the complex scalar functions Φ and Ψ, which serve as potentials for **K**. $\hat{\mathbf{n}}$ is normal to S. ∇_S is the intrinsic (to S) gradient operator.

The resulting equation is obtained by multiplying (1) with e^{ikz}, and then use (2) to express everything in terms of Φ and Ψ. The resulting equation is not given here, see for instance [3]. Rather, we focus on possible advantages and results. Two major advantages are the facts that 1) After multiplication with e^{ikz}, the left hand side of (1) becomes an exact one-form (and this is true whether on regards $-E_0\hat{\mathbf{x}}$ as a one-form in $\mathbf{R}^3$ or as a one-form on S), and 2) The replacement $\mathbf{K}(\mathbf{r}') = e^{ikz'}\mathbf{J}(\mathbf{r}')$ allows for potentially sparser sampling, and hence reduced numerics. (C.f. [5].)

3 Numerical results

We have performed calculations on a 100° dihedral with dimensions as in Fig. 1. The calculations have

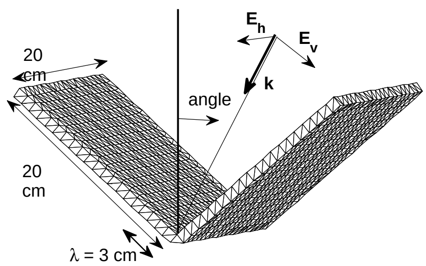

Fig. 1. The dihedral with its dimensions. The opening angle of the dihedral is 100 °, and the dihedral is illuminated with a plane wave with f=10 GHz from above at different angles. The convention for horizontal and vertical polarization is indicated in the figure.

been performed with different meshes, resulting in a simple convergence study. The calculations are also compared with measurements and finally the number of unknown are compared to the number of unknown suggested by a commercial software. In Fig. 2, calcu-

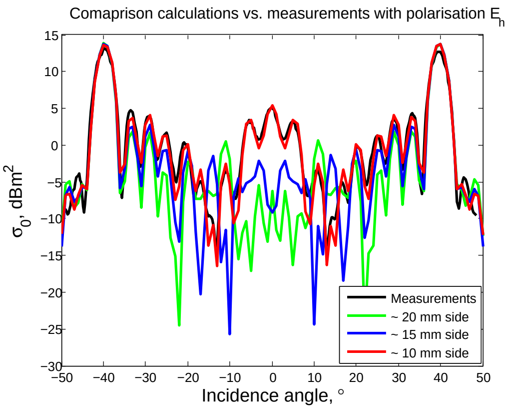

Fig. 2. Measurements vs. calculations for different mesh sizes, horizontal polarization (see Fig. 1). Numbers in the legend indicate typical side length in the mesh.

lations are compared with measurement for horizontal polarization (see Fig. 1). A reasonable agreement with measurements and calculations are obtained with a mesh size with a typical side length of 10 mm. Compared with the wavelength λ, this is only $\sim$three triangles/wavelength which is well below the rule of thumb which is typically eight or ten triangles/wavelength. With our mesh, we have 3406 triangles and a total number of unknown which is 3410. Using the commercial software FEKO, (in standard MoM setting), it is for the given geometry and frequency suggested a mesh which gives 127000 unknowns. Although we do not claim our results to be as accurate as with FEKO, the reduction of unknowns is substantial. On the other hand, using the same meshes for calculations of the scattering from vertical polarization, the results are less satisfactory, see Fig. 3. By refining the mesh, clear improvements are noticed for the mesh with a side length of $\sim$5 mm, especially around incidence angle around 0°, although the agreement is worse around incidence angles around 25°. It might

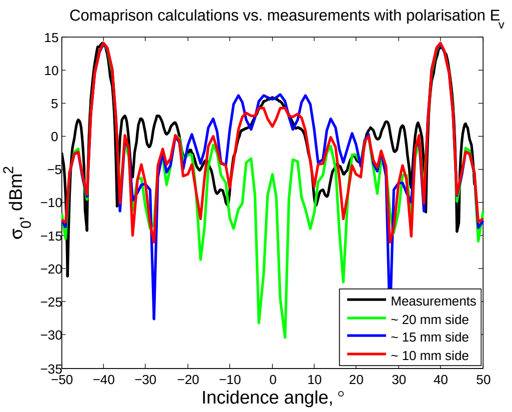

Fig. 3. Measurements vs. calculations for different mesh sizes, vertical polarization (see Fig. 1). Numbers in the legend indicate typical side length in the mesh.

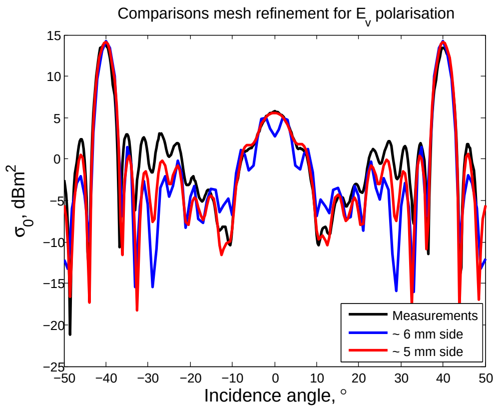

Fig. 4. Measurements vs. calculations for mesh refinements, vertical polarization (see Fig. 1). Numbers in the legend indicate typical side length in the mesh.

be claimed that this mesh size is close to the rule of thumb, but the mesh is only refined at the illuminated part, with parts in the shadow having a coarser mesh. As a result, the number of unknown N is 16872, which is still a good factor less than 127000. As the cost for solving the resulting linear equation scales as N^3, there is a noticeable difference.

4 Conclusions

We have applied the potential method for calculations on a dihedral with opening angle of 100°. It is indicated that the non-convexity of the dihedral requires different mesh sizes in different polarizations. However, in both cases, reasonable results are produced when the number of unknowns are well below the number of unknown given by meshes following the rule of thumb, saying that the side lengths should be $\sim \lambda/10$. This decreases the memory requirements as well as the time for solving the produced linear system, as compared to standard MoM.

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