Reduced order modeling of ODE-PDE networks

Michael Hinze[1] and Ulrich Matthes[1]

Department of Mathematics, University of Hamburg, Bundesstr. 55, 20146 Hamburg, Germany,
michael.hinze@uni-hamburg.de, ulrich.matthes@math.uni-hamburg.de

Summary. We propose a model order reduction (MOR) approach for networks containing simple and complex components modeled by linear ODE and nonlinear PDE systems respectively. These systems are coupled through the network topology using the Kirchhoff laws. We consider as application MOR for electrical networks, where semiconductors form the complex components. POD combined with discrete empirical interpolation (DEIM) and passivity-preserving balanced truncation methods for electrical circuits (PABTEC) can be used to reduce the dimension of the whole model.

1 Introduction

We propose a simulation-based MOR approach for the reduction of networks consisting of (many) simple and (only few) complex components. We assume that the simple and complex components are modeled by systems of linear ODEs and nonlinear PDEs, respectively, which are coupled through the network topology using the Kirchhoff laws.

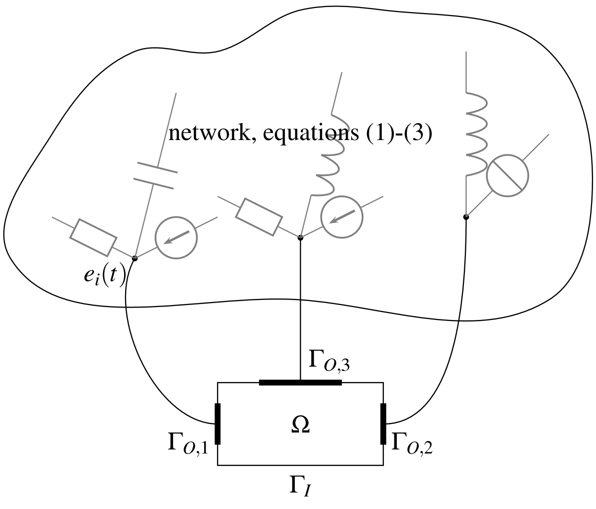

Fig. 1. Sketch of a coupled system with one semiconductor forming the complex component.

We consider electrical networks where the simple components consist of resistors, capacitors, voltage sources, current sources, and inductors, and the complex components are formed by e.g. semi-conductors, see Fig. 1. The overall system then is represented by a nonlinear PDAE system, see e.g. [2, 5]. We address the following issues:

1. construction of reduced order models for the complex components
2. reduction of the complete network while retaining the structure of a network

2 Modeling of an electrical network

In electrical networks resistors, capacitors, and inductors form the simple components which in general are modeled by linear ODEs. Complex components are given by e.g. semiconductors which are modeled by PDE systems. Considering additional voltage and current sources the overall network can be modeled by a partial-differential algebraic equation (PDAE) which is obtained as follows. First the network containing only the simple components is modeled by a differential algebraic equation (DAE) which is obtained by a modified nodal analysis (MNA), including the Ohmic contacts Γ_O of the semiconductors as network nodes, see Fig. 1. Denoting by e the node potentials and by j_L, j_V, and j_S the currents of inductive, voltage source, and semiconductor branches, the DAE reads (see [5, 9, 12])

$$A_C \frac{d}{dt} q_C(A_C^\top e, t) + A_R g(A_R^\top e, t) + A_L j_L + A_V j_V + A_S j_S = -A_I i_s(t), \qquad (1)$$

$$\frac{d}{dt}\phi_L(j_L, t) - A_L^\top e = 0, \qquad (2)$$

$$A_V^\top e = v_s(t). \qquad (3)$$

Here, the incidence matrix $A = [A_R, A_C, A_L, A_V, A_S, A_I]$ represents the network topology, e.g. at each non mass node i, $a_{ij} = 1$ if the branch j leaves node i and $a_{ij} = -1$ if the branch j enters node i and $a_{ij} = 0$ elsewhere. In particular the matrix A_S denotes the semiconductor incidence matrix. The functions q_C, g and ϕ_L are continuously differentiable defining the voltage-current relations of the network components. The continuous functions v_s and i_s are the voltage and current sources. For details we refer to [7].

In a second step the semiconductors are modeled by PDE systems, which are then coupled to the DAE of the network, see e.g. [1, 2] and the references cited there. Further details of our approach are given in [7]. The analytical and numerical analysis of PDAE systems of the presented form is subject to current research, see [2, 4, 11, 12].

3 Reduced order models for complex components

We assume that every complex component is modeled by a time-dependent PDE system which is amenable to a numerical treatment with Galerkin methods. After appropriate spatial discretization the method of lines then yields a large, nonlinear ODE system representing the spatially discrete complex component. This nonlinear ODE system now represents the complex component in the network. The reduction of the complex components is based on simulation-based MOR with proper orthogonal decomposition (POD). In this approach time snapshots of the complex components are extracted from snapshots of the simulation of the complete network. POD for the complex component then is performed using the extracted parts of the snapshots. In combination with the direct empirical interpolation method (DEIM) this now delivers low dimensional, nonlinear surrogate models for the complex components, see [6] for details. It is an important feature of this reduction technique that it delivers distinct reduced order models for the same complex component at different locations in the network.

4 Reduction of the whole network

The overall network with simple and complex components is represented by a nonlinear DAE system, where the linear and nonlinear part stems from the simple and spatially-discrete complex components respectively. The reduction for the complex components is performed as in the previous section, whereas the linear part is approximated by a reduced order linear model of lower dimension. In the case of an electrical network the passivity preserving reduction method (MATLAB Toolbox) PABTEC [8, 10] is used for the reduction of the linear part of the network. Finally, the reduced order models obtained with the approaches sketched are recoupled appropriately. The obtained large and sparse nonlinear DAE system as well as the small and dense reduced-order model are integrated using the DASPK software package [3] based on a BDF method, where the nonlinear equations are solved using Newton's method.

The results obtained demonstrate that the recoupling of the PABTEC reduced order model with the POD-MOR model for the semiconductor delivers an overall reduced-order model for the circuit-device system which allows significantly faster simulations (the speedup-factor is about 20) while keeping the relative errors below 10%.

Finally we sketch how our approach can be applied to parametrized MOR extending the techniques of [7].

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