Efficient Simulation of Frequency-Transient Mixed Co-Simulation of Coupled Heat-Electromagnetic Problems

Christof Kaufmann[1], Michael Günther[1], Daniel Klagges[2], Jan ter Maten[1], Matthias Richwin[2], Sebastian Schöps[1]

[1] Bergische Universität Wuppertal, Wicküler Park, Gaußstraße 20, D-42119 Wuppertal
{kaufmann,schoeps,guenther,termaten}@math.uni-wuppertal.de
[2] Leopold Kostal GmbH & Co. KG, Hauert 11, 44227 Dortmund, {d.klagges,m.richwin}@kostal.com

Summary. This paper discusses an efficient mixed mode simulation method for induction heating problems. For time-harmonic inputs the electromagnetic part can be considered in the frequency domain. This avoids the inefficient time integration of high frequency signals. By leaving the heat problem in time domain this approach leads to a frequency-transient simulation with low computational costs. The coupling is established by an average power transfer model.

1 Introduction

Inductive power transfer problems deal with electromagnetic fields that transport large amounts of energy. Even small power losses can result in considerable heating up [3, 5, 6]. The skin effect causes most heat generation at material boundaries. Hence the temperature considerably influences the material parameters, e.g. the electric conductivity, and thus the electromagnetic fields. This underlines the need for a mutual coupling of the heat and electromagnetic field models.

A transient simulation of the coupled problem often suffers from relative small time steps due to high frequencies in the electromagnetic part. However, heating up is a comparatively slow effect. Therefore simulations of large time intervals are necessary. The small time steps in combination with long time intervals induce high computational cost or make a simulation even infeasible (multirate behaviour).

It is beneficial to reduce the computational effort for solving the electromagnetic problem. In this paper we discuss an adapted model that allows for a mixed formulation: frequency domain analysis of the EM problem and time domain for the heat problem, [2]. This approach is similarly implemented in COMSOL Multiphysics, [4]. We focus on numerical analysis in the framework of dynamic iteration, e.g. [1].

An model example from industry is used for numeric results. KOSTAL describes with that the power transfer by induction for an inductive charging station. It will be used to charge batteries of electric cars.

2 Modelling

Electromagnetic fields are mathematically described by a system of time-dependent partial-differential equations on a domain Ω. It reads in curl-curl formulation:

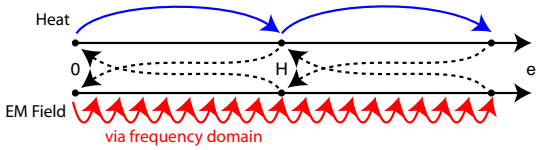

Fig. 1. Time windows τ_i, time steps due to dynamics (red and blue arrows) and coupling scheme (black arrows). Small time steps (red) are avoided in frequency domain.

$$\varepsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} + \sigma(T) \frac{\partial \mathbf{A}}{\partial t} + \nabla \times (\nu \nabla \times \mathbf{A}) = \mathbf{J}_\mathrm{e}, \qquad (1)$$

where the reluctivity ν and the permittivity ε depend only on space $\mathbf{r} \in \Omega$, the electrical conductivity σ also on temperature T, the external current density $\mathbf{J}_\mathrm{e}$ is a given sinusoidal source and the magnetic vector potential $\mathbf{A}(t)$ is unknown on $t \in [t_0, t_e]$. For brevity the space dependency $\mathbf{r}$ is always neglected. On the other hand we have the heat equation

$$\rho\, c \frac{\partial T}{\partial t} = \nabla \cdot (\mathbf{k} \nabla T) + Q, \qquad (2)$$

where the temperature T is an unknown function and the mass density ρ, the heat capacity c and the heat conductivity $\mathbf{k}$ depends on space and temperature. The term $\mathbf{Q}$ is a source term. It is given by the power loss of the electromagnetic field and couples (1) and (2). If we neglect hysteresis losses, Q is described by

$$Q(\mathbf{A}, T) = \sigma(T) \frac{\partial \mathbf{A}}{\partial t} \cdot \frac{\partial \mathbf{A}}{\partial t} - \frac{\partial \mathbf{A}}{\partial t} \cdot \mathbf{J}_\mathrm{e}\,. \qquad (3)$$

We equip (1)-(3) with boundary and initial conditions at t_0 and discretise it. However (1) requires very small time steps for the fast varying signal $\mathbf{J}_\mathrm{e}$. This problem is addressed in the next section.

3 Averaging Power and Temperature

We split the time interval of interest $[t_0, t_e]$ in time windows $[\tau_i, \tau_{i+1}]$ according to the time scale of the heat transfer, see Fig. 1. Since heat transfer is a rather slow process, it is sufficient to consider only the averaged power per time window that is generated:

$$\bar{Q}_i = \frac{1}{\tau_{i+1} - \tau_i} \int_{\tau_i}^{\tau_{i+1}} Q(\mathbf{A}(t), T(t))\, dt \qquad (4)$$

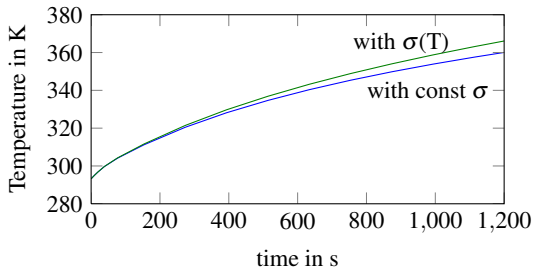

Fig. 2. Results with temperature dependent σ and const. σ

and similarly the temperature is averaged:

$$\bar{T}_i = \frac{1}{\tau_{i+1} - \tau_i} \int_{\tau_i}^{\tau_{i+1}} T(t)\, dt. \tag{5}$$

It follows for time-harmonic input signals $\mathbf{J}_\mathrm{e}$

$$\bar{Q}_i = \sigma(\bar{T}_i)\, \frac{\omega^2}{2} \left\| \hat{\mathbf{A}}_c \right\|_c^2 + \frac{\omega}{2} \mathrm{Im}(\hat{\mathbf{A}}_c) \cdot \hat{\mathbf{J}}_\mathrm{e}, \tag{6}$$

where ω is the angular frequency and $\hat{\mathbf{J}}_\mathrm{e}$ the amplitude of $\mathbf{J}_\mathrm{e}$ and $\hat{\mathbf{A}}_c$ is the complex fourier coefficient of the solution for $\mathbf{A} = \hat{\mathbf{A}}_c\, \mathrm{e}^{j\omega t}$ of (1) with $\mathbf{J}_\mathrm{e} = \hat{\mathbf{J}}_\mathrm{e}\, e^{j\omega t}$.

We derive a simplified system consisting of a (1) in frequency domain and a (2) in time domain:

$$(j\,\omega\,\sigma(\bar{T}_i) - \omega^2\,\varepsilon)\hat{\mathbf{A}}_c + \nabla \times (\nu\,\nabla \times \hat{\mathbf{A}}_c) = \hat{\mathbf{J}}_\mathrm{e} \tag{7}$$

$$\rho\, c\, \frac{\partial T}{\partial t} - \nabla \cdot (\mathbf{k} \nabla T) = \bar{Q}_i, \tag{8}$$

where $\bar{Q}_i$ is defined in (6) and $\bar{T}_i$ in (5). Equation (7) is equivalent to an average power transfer model of (1). However, in frequency domain only a linear system has to be solved instead of many time steps. This approach exploits efficiently different time scales.

4 Co-simulation

We solve the system (6)-(8) iteratively, [1], In the following the subscript index i belongs to time step t_i and the superscript index (l) denotes the iteration step l.

$$(j\omega\,\sigma(\bar{T}_{i+1}^{(l)}) - \omega^2\,\varepsilon)\hat{\mathbf{A}}_{i+1}^{(l+1)} + \nabla \times (\nu\,\nabla \times \hat{\mathbf{A}}_{i+1}^{(l+1)}) = \hat{\mathbf{J}}_\mathrm{e}$$

$$\bar{Q}_{i+1}^{(l+1)} = \sigma(\bar{T}_i^{(l+1)})\, \frac{\omega^2}{2} \left\| \hat{\mathbf{A}}_{i+1}^{(l+1)} \right\|_c^2 + \frac{\omega}{2} \mathrm{Im}(\hat{\mathbf{A}}_{i+1}^{(l+1)}) \cdot \hat{\mathbf{J}}_\mathrm{e}$$

$$\bar{T}_{i+1}^{(l+1)} - \frac{h_i}{\rho\, c} \nabla \cdot (\mathbf{k} \nabla \bar{T}_{i+1}^{(l+1)}) = \bar{T}_i + \frac{h_i}{\rho\, c} \bar{Q}_{i+1}^{(l+1)}$$

The co-simulation can be organized as shown in Fig. 3 for the special case where time step and time window sizes agree, i.e., $h_i = \tau_{i+1} - \tau_i$.

In the full paper this algorithm is numerically analysed and convergence of the inner loop is shown. This converges to the *average power and temperature model* from Sec. 3. In a second step it will be shown, that this model converges to the original model from Sec. 2 when the time steps turn to zero. In addition the computational sequence of the subsystem will be discussed. The results are verified by a 2D model of the industry example, see Fig. 2.

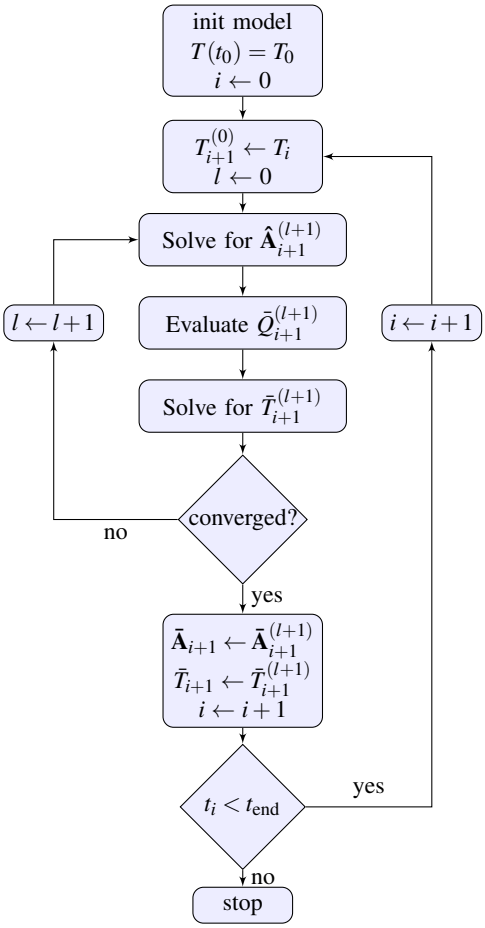

Fig. 3. Co-Simulation

Acknowledgement. This work is supported by the German BMBF in the context of the SOFA project (grant number 03MS648E).

References

1. A. Bartel, M. Brunk, M. Günther, and S. Schöps. Dynamic iteration for coupled problems of electric circuits and distributed devices. Preprint 11/16, 2011.
2. J. Driesen and K. Hameyer. The simulation of magnetic problems with combined fast and slow dynamics using a transient time-harmonic method. *Eur Phys J Appl Phys*, 14:165–169, 2001.
3. H.H.J.M. Janssen, E.J.W. ter Maten, and D. van Houwelingen. Simulation of coupled electromagnetic and heat dissipation problems. *IEEE Trans Magn*, 30(5):3331 –3334, sep 1994.
4. COMSOL Multiphysics. *Command Reference*. Comsol AB, 2007.
5. E.J.W. ter Maten and J.B.M. Melissen. Simulation of inductive heating. *IEEE Trans Magn*, 28(2):1287 –1290, mar 1992.
6. J. Will. An optimal control problem in electromagnetic induction heating. Master's thesis, Chemnitz University of Technology, Department of Mathematics, Germany, 2010.