Electromagnetic Eigenmode Characterization by Sensitivity Analysis

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Summary. Computational models of resonant electromagnetic structures which are bounded by perfectly matched layers have an eigenvalue spectrum which is spoilt by eigenmodes which reside within these layers. In the context of the finite integration technique we apply a computational inexpensive sensitivity analysis in order to identify those undesired eigenmodes.

1 Motivation

The computation of electrodynamic eigenmodes of radiating structures is a challenging task, since the transition to free-space at the boundaries has to be modeled. An established technique to model that transition is the use of a *perfectly matched layer* (PML) [5]. The PML causes the eigenvalue problem of Maxwell's equations to become complex for structures of any material. Moreover, the PML consists of artificial materials whose parameters can be large in magnitude, which causes some eigenmodes to be *trapped* within the PML. In this contribution we show an computationally efficient analysis which is based on the eigenvalues' sensitivity that is able to decide whether a specific eigenmode is bound to the PML or the structure. The approach follows an adjoint technique which is known since quite some time [2, 3]. Recent advances considering the sensitivity analysis of waveguide models has been shown in [1].

2 Computational Approach

The discrete Maxwell's eigenvalue problem is set up in the framework of the finite integration technique (FIT) [4]. The Maxwell grid equations can be written down in frequency domain, neglecting currents and charges, for dispersive materials as

$$\mathbf{C}\widehat{\mathbf{e}} = -s\mathbf{M}_\mu(s)\widehat{\mathbf{h}}, \qquad \mathbf{C}^T\widehat{\mathbf{h}} = s\mathbf{M}_\varepsilon(s)\widehat{\mathbf{e}}, \tag{1}$$

where $\mathbf{C} \in \mathbb{R}^{N\times N}$ is the topological curl-operator consisting of entries with $\{-1;0;1\}$ and $s = \mathrm{i}\omega = 2\pi\mathrm{i}f$ is the frequency. The constitutive relations read

$$\widehat{\widehat{\mathbf{d}}} = \mathbf{M}_\varepsilon(s)\widehat{\mathbf{e}} \quad \text{and} \quad \widehat{\widehat{\mathbf{b}}} = \mathbf{M}_\mu(s)\widehat{\mathbf{h}}. \tag{2}$$

An absorbing boundary condition based on complex metric stretching perfectly matched layer (PML) [5] can be introduced in FIT in a straight-forward manner. Since the PML is only in the continuous case *perfectly matched* a remaining reflection error is introduced that can be controlled by the number and the step width of the absorbing layers. The introduction of dielectric and magnetic losses in the PML cause the diagonal material matrices $\mathbf{M}_\mu$ and $\mathbf{M}_\varepsilon$ to become complex. The actual frequency dependency of the components of the material matrices on the PML parameters reads exemplarily for the permeability

$$\mu^{-1}(s) = \frac{1+\frac{\sigma_n}{s}}{1+\frac{\sigma_{t1}}{s}+\frac{\sigma_{t2}}{s^2}}\,\mu_0^{-1}. \tag{3}$$

In frequency domain we solve the curl-curl eigenmode equation for complex resonance frequencies $-s^2$ and grid-voltages $\widehat{\mathbf{e}}$, which are derived from (1) as

$$\mathbf{A}(s)\widehat{\mathbf{e}} = -s^2\widehat{\mathbf{e}}, \quad \mathbf{A}(s) = \mathbf{M}_{\varepsilon^{-1}}(s)\mathbf{C}^T\mathbf{M}_{\mu^{-1}}(s)\mathbf{C}. \tag{4}$$

At this point the eigenvalue problem (4) has a polynomial-type nonlinearity. Since the PML is designed to operate quite well over a certain frequency range, the frequency dependent material matrices are evaluated at the estimation frequency s_{est}, in order to linearize the eigenvalue problem (4). Yet, the system matrix $\mathbf{A}(s)$ remains complex with eigenvalues $-s^2$. The solution can be computationally expensive, but yields the modal field distributions as well as their resonance frequency and quality factors $Q = \Im\{s\}/2\Re\{s\}$. Moreover, the spectrum is spoilt by undesired modes, which are trapped within the PML and occur at similar frequencies like the desired modes.

3 Eigenvalue Sensitivity Analysis

We start with a complex eigenvalue problem of the type $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$ and its derivative

$$(\mathbf{A}' - \lambda'\mathbf{I})\mathbf{x} + (\mathbf{A} - \lambda\mathbf{I})\mathbf{x}' = 0. \tag{5}$$

The primed quantities denote derivations with respect to the design parameter p e.g. $\mathbf{A}' := \partial\mathbf{A}/\partial p$. Following the standard perturbation theory [2] the multiplication from the left with the corresponding left eigenvector $\mathbf{y}^H$ and substitution of $\mathbf{y}^H\mathbf{A} = \lambda\mathbf{y}^H$ (the definition of the left eigenvalue problem) finally yields the derivative of the eigenvalue

$$\lambda' = \frac{\mathbf{y}^H \mathbf{A}' \mathbf{x}}{\mathbf{y}^H \mathbf{x}}, \qquad (6)$$

which could be further simplified, if the left and right eigenvectors were orthonormalized. The left eigenvectors $\mathbf{y}^H$ of a matrix eigenvalue problem $\mathbf{y}^H \mathbf{A} = \lambda \mathbf{y}^H$ can be computed as the right eigenvectors of the matrix' adjoint $\mathbf{A}^H \mathbf{y} = \lambda^* \mathbf{y}$, where * denotes the complex conjugate.

4 Application to an Example in the FIT

The FIT system matrix $\mathbf{A}$ from (4) can be made complex-symmetric by a similarity transform with $\mathbf{M}_{\varepsilon^{-1/2}}$. The adjoint of the complex-symmetrized matrix satisfies

$$\mathbf{A}_{sym}^H = \mathbf{A}_{sym}^*, \qquad (7)$$

which is simply the complex-conjugate matrix. Eigenvectors of $\mathbf{A}^H$ are identified as the dielectric grid fluxes $\widehat{\widehat{\mathbf{d}}}^*$. However, instead of solving the eigenvalue problem itself we can get the dielectric grid fluxes simply from the matrix-vector multiplication given in the material relation (2).

Figure 1a shows the structure for our numerical tests, which consists of a small dielectric slab having $\varepsilon_r = 5$ in a parallel-plate waveguide. An undesired as well as an desired eigenmode are included in Fig. 1b and 1c respectively. The lateral boundaries are modeled by a PML.

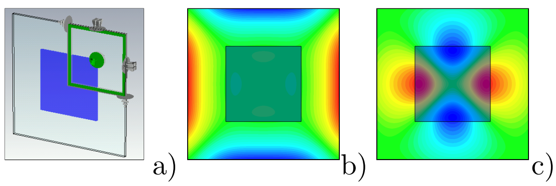

Fig. 1. a) Model of dielectric square having $\varepsilon_r = 5$. b) Undesired eigenmode at 13.76 GHz. c) Structure eigenmode at 12.82 GHz.

In Fig. 2 the loci of eigenvalues are plotted for different values of the linearization parameter s_{est}. It turns out that eigenvalues which are weakly dependent on s_{est} are those of structure eigenmodes (∘). For sensitivity analysis the frequency dependent system matrix is derived by s_{est}.

Figure 3 shows the magnitude $|\lambda'|$ obtained by (6). Again small values belong to eigenmodes whose field distribution (cf. Fig. 1c) is primarily concentrated within the structure (∘). Eigenmodes whose field distribution is contained within the PML show a magnitude of $|\lambda'|$ which is larger than zero. The absolute limits for decisions on $|\lambda'|$ are the topic for further investigations.

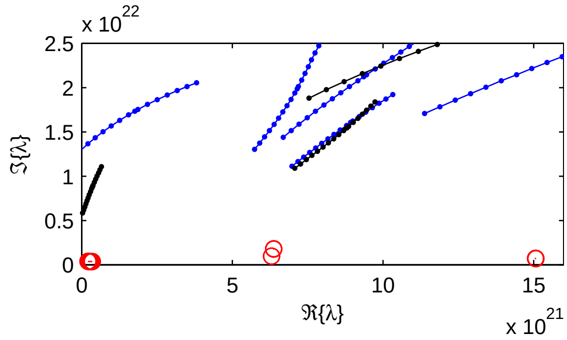

Fig. 2. Loci of eigenvalues λ for linearization parameters $s_{est} \in [2\pi\mathrm{i}\cdot 8\ \mathrm{GHz}, 2\pi\mathrm{i}\cdot 16\ \mathrm{GHz}]$. Neglectable deviations of the data sets indicate eigenfrequencies with a field distribution within structure and low PML dependency (∘).

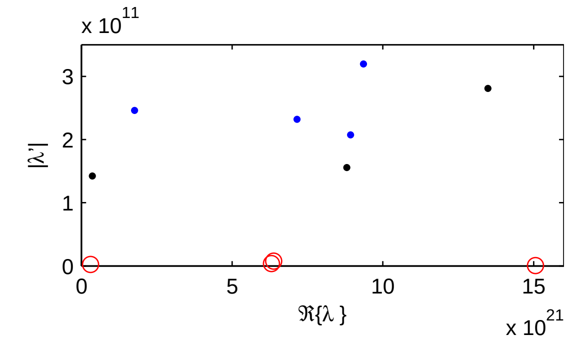

Fig. 3. Magnitude of the derivative $|\lambda'|$ over $\Re\{\lambda\}$, linearized at $s_{est} = 2\pi\mathrm{i}\cdot 12.83$ GHz. Small values indicate eigenmodes that are bound to the dielectric substructure (∘).

5 Conclusion

We present a methodology which is able to decide which eigenmode belongs originally to the computational model and which is introduced by the perfectly matched layers absorbing boundary condition.

References

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