A new indicator to assess the quality of a Pareto approximation set applied to improve the optimization of a magnetic shield.

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Summary Evaluating the performances of an optimization algorithm is more complex in the case of multi-objective optimization problems than single-objective ones. In the former case, the optimization aims to obtain a set of non-dominated solutions close to the Pareto-optimal front, well-distributed, maximally extended and full filled. This paper presents a new quality indicator encompassing the aforementioned goals. The quality indicator is then used to select a suitable algorithm for the multi-objective optimization of a magnetic shield in an induction heating system.

1 Introduction

The optimization results provided by a multi-objective algorithm are, usually, a set of non-dominated solutions (called approximation set in the decision space and Pareto approximation front in the objective functions space).

The main goal of such algorithms is to provide an approximation set matching the Pareto-optimal set.

The notion of performance of an optimization algorithm involves the quality of the solutions that it is able to produce and the computational effort required to provide such solutions.

The definition of quality is a complex topic to deal with in the case of multi-objective optimization problems. A good optimization algorithm should [1]:

- minimize the distance from the Pareto approximation front to the Pareto-optimal front;
- obtain a good (usually uniform) distribution of the solutions found;
- maximize the extension of the Pareto approximation front, i.e., for each objective, a wide range of values should be covered by the non-dominated solutions;
- maximize the “density” of the Pareto approximation front, i.e. is desirable a high cardinality for the approximation set.

In literature, there are different methods that assign a quality indicator or a set of quality indicators that are a measure of the aforementioned goals and, usually, a combination of them is used in order to evaluate the goodness of a multi-objectives optimization algorithm [2].

In this paper, a new unary quality indicator, called *Degree of Approximation* (*DOA*), is presented. It takes into account all the goals listed before. DOA was then helpful for the choice of the optimization algorithm more suitable to perform the multi-objective optimization of a magnetic shield.

2 Degree of Approximation indicator

DOA is a distance-based unary quality indicator that also encompasses the distribution, the extension and the cardinality of a Pareto approximation front.

In detail, for a Pareto front approximation set A, *DOA* is computed as described in the following.

First, given a solution i belonging to the Pareto-optimal front (POF), the sub-set of A containing the solutions dominated by i, $D_{i,A}$, is determined. Hence, if the number of elements belonging to $D_{i,A}$ is not null ($|D_{i,A}|>0$), for each approximated solution $a \in D_{i,A}$ is computed the Euclidean distance $df_{i,a}$ (see Fig.2) between a and i as:

$$df_{i,a} = \sqrt{\sum_{k=1}^{n} \left[f_{k,a} - f_{k,i} \right]^2} \qquad (1)$$

where n is the number of objective functions, $f_{k,a}$ is the value of the k-th objective function of approximated solution a, $f_{k,i}$ is the value of the k-th objective function of optimal solution i.

Then the parameter $d_{i,A}$ is computed: it is the Euclidean distance between i and the nearest approximated solution belonging to $D_{i,A}$:

$$d_{i,A} = \begin{cases} \min\left(df_{i,a}\right) & a \in D_{i,A} \quad if\ \left|D_{i,A}\right| > 0 \\ \infty & \quad if\ \left|D_{i,A}\right| = 0 \end{cases} \qquad (2)$$

Another quantity, $rf_{i,a}$ is computed as:

$$rf_{i,a} = \sqrt{\sum_{k=1}^{n} \left[\max\left(0,\, f_{k,a} - f_{k,i}\right)\right]^2} \qquad (3)$$

it is a 'reduced' distance between i and a not dominated solution a of A.

Then considering the solutions of A not dominated by i. the parameter $r_{i,A}$, is computed similarly to $d_{i,A}$:

$$r_{i,A} = \begin{cases} \min\left(rf_{i,a}\right) & a \in A \setminus D_{i,A} \quad if \ \left|A \setminus D_{i,A}\right| > 0 \\ \infty & if \ \left|A \setminus D_{i,A}\right| = 0 \end{cases} \qquad (4)$$

Finally, defining, for each $i \in$ POF, the value $s_{i,A}$ as the minimum between $d_{i,A}$ and $r_{i,A}$, the new unary quality indicator, *DOA*, is computed as:

$$DOA\,(A) = \frac{1}{|POF|} \sum_{i=1}^{|POF|} s_{i,A} \qquad (5)$$

The smaller is DOA the better is the Pareto approximation front.

3 Optimization of a magnetic shield

The DOA quality indicator was used to compare the Pareto approximation fronts given by two different multi-objective optimization algorithms NPAEP [3] and NSGA II [4] applied to mathematical benchmark problems for which the true POF was known. In particular, Table 1 shows the results obtained by NPAEP and NSGA II for the Fonseca and Fleming problem [5] (FON) using only 1000 and 2500 objective function evaluations. The results are the DOA mean values (over one hundred trials): the lower the index is the better the algorithm works.

Table 1. Results for the FON problem.

n_v	NPAEP	NSGA II
1000	0.006427	0.018121
2500	0.003008	0.006400

NPAEP works better than NSGA II, it is worth pointing out that NSGA II needs 2500 fitness evaluations to reach results comparable to those of NPAEP. For all the mathematical benchmark problems with few design variables and for which the true POF was regular, NPAEP shown the same behaviour. NSGA II outperforms NPAEP when the number of design parameters increases.

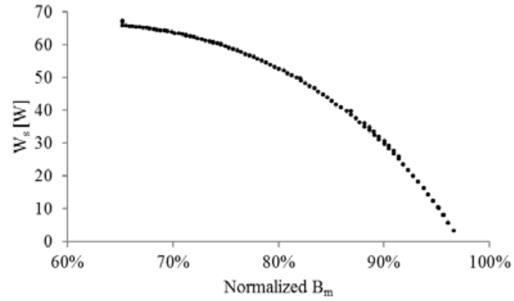

Fig. 1. POF provided by NPAEP.

Hence NPAEP was chosen for the optimization of the shielding of the axisymmetric induction heating system optimized in [6] in which the different objectives were combined in a single objective function. Two design parameters are used: the semi-height and the outer radius of the steel shield; while the two optimization targets to minimize, i.e. the mean magnetic induction B_m in the target area and the power losses W_s in the shield, are kept distinct. Here are reported the results of the passive shield optimization only. Figure 1 shows the POF obtained by NPAEP after 2000 numerical simulations carried out by means of FEM-DBCI [6]. The POF is well-distributed and full filled thus the decision maker has several solutions to choose from.

More details and results will be given in the full contribution.

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