Body-fitting meshes for the Discontinuous Galerkin Method

J. Cui[1], S. M. Schnepp[1], and T. Weiland[1, 2]

[1] Graduate School of Computational Engineering, Technische Universitaet Darmstadt, Dolivostrasse 15, 64293 Darmstadt, Germany, cui@gsc.tu-darmstadt.de, schnepp @gsc.tu-darmstadt.de
[2] Institut fuer Theorie Elektromagnetischer Felder, Technische Universitaet Darmstadt, Schlossgartenstrasse 8, 64289 Darmstadt, Germany, thomas.weiland@temf.tu-darmstadt.de

Summary. A mesh scheme is developed to deal with curved boundaries of the geometry using quadrilateral elements for the Discontinuous Galerkin Method (DGM). To achieve this, we first generate the inner part of the mesh in a structured manner and connect it to the curved boundary with a so-called buffer layer. Elements in the buffer layer employ a high order mapping to fit the boundary. We demonstrate high order convergence rates with an electromagnetic problem in a cylindrical cavity. Furthermore, we show that the frequency spectrum, which is extracted from the time-domain signal is clean, i.e., no spurious modes are observed in any of the examples considered.

1 Introduction

The DGM is a high order numerical method. In order to maintain its high order accuracy in the presence of curved objects, boundaries (surfaces) of the geometries have to be described with high order accuracy as well. The study in [1] shows that meaningful high order accurate results can be obtained only if the curved boundaries are considered with high order geometric approximations. In [2] problems in a cylindrical cavity are solved by pushing the straight edges of elements onto the exact circular boundary.

Both implementations [1, 2] employ triangular meshes for the DGM and achieve high order convergence. We propose an alternative mesh scheme based on Cartesian grids. It generates quadrilateral meshes in a simple process for both, exact geometries and objects represented by Non-Uniform Rational B-Splines (NURBS). The scheme enjoys many advantages due to the ability of applying tensor product bases within quadrilateral elements (see e.g. [3, 4]).

2 Body-fitting mesh scheme

We generate a set of buffer elements in the gap between the exact curved boundary and the interior structured mesh as demonstrated in Fig. 1. Figure 2 (left) shows that if no buffer layer is applied, degenerated elements (marked with arrows) are likely to occur , which is guaranteed not to happen with the insertion of a buffer layer [5] (right). Figure 3 gives an example, where a mesh is generated fitting a geometry described by NURBS. For performing the local element deformation in the buffer layer we apply Transfinite Interpolation (TFI) [6].

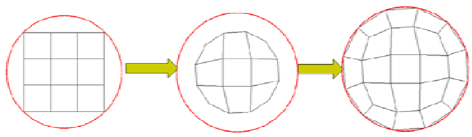

Fig. 1: Buffer layer mesh scheme based on a 3-by-3 regular mesh.

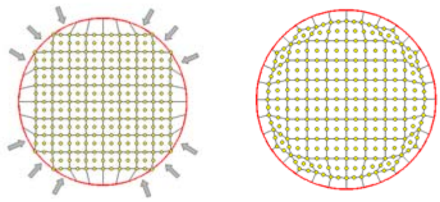

Fig. 2: Curved elements of 2nd order without (left) and with (right) buffer layer scheme based on a 9-by-9 regular mesh.

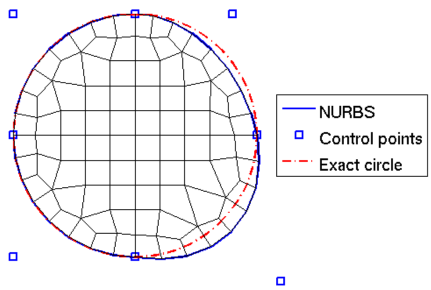

Fig. 3: Buffer layer mesh with NURBS. The approximation can be exact for both a circle (left half) and an arbitrary curve (right half) using control points.

3 Solving electromagnetic problems

We consider transverse magnetic (TM) problems in a two-dimensional circular domain Ω with the boundary $\partial\Omega$. The Maxwell's equations read as follows:

$$\mu\frac{\partial H_x}{\partial t} = -\frac{\partial E_z}{\partial y}, \quad \mu\frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x}, \qquad (1)$$

$$\varepsilon\frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}, \qquad (2)$$

where H_x and H_y are the *x*- and *y*-components of the magnetic field vector, and E_z the *z*-component of the electric field vector. The parameters ε and μ are the electric permittivity and the magnetic permeability, respectively.

In this DGM approach, Legendre polynomials are applied as basis functions and the explicit leap-frog scheme is used for the time discretization [4]. The TM31 mode in a cylindrical cavity is chosen for a convergence study. The errors are measured in the L^2 norm at the end of one periodic oscillation.

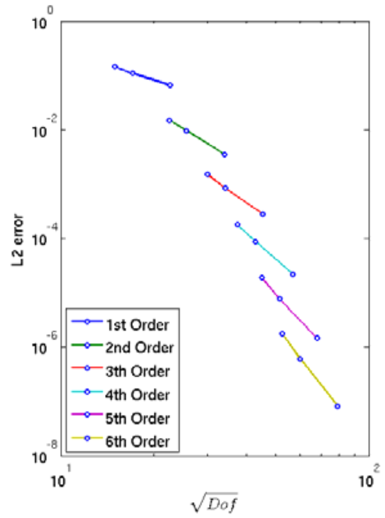

Fig. 4: For a resonant mode in the cylindrical cavity, DGM with upwind flux shows (*p*+1) convergence using body-fitting meshes.

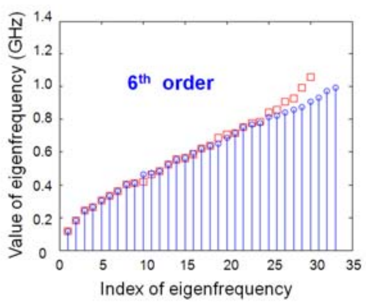

Fig. 5: Analytical values (red squares) and captured numerical eigen-modes (blue stems)

Figure 4 shows that the optimal convergence of (*p*+1) is achieved where *p* is the polynomial order. We also extracted eigenfrequencies via a Fourier Transform. The results in Fig. 5 were obtained using central fluxes and 32 elements of 6th order. The eigenfrequencies obtained from the time-domain solution agree with the analytical ones for frequencies up to 0.8 GHz. Above this frequency the spatial resolution is insufficient leading to errors.

4 Conclusions

A body-fitting mesh scheme employing high order curved elements with the DG method is proposed. High order convergence rates in the presence of curved objects are observed. Furthermore, we extracted frequency spectra from simulations of a cylindrical cavity and found the agreement between the numerical results and the respective analytical solutions, i.e., clean spectra are obtained.

Acknowledgement. The work of J. Cui and S. M. Schnepp is supported by the 'Initiative for Excellence' of the German Federal and State Governments and the Graduate School of Computational Engineering at Technische Universitaet Darmstadt.

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