On the autocorrelation of environment induced noise

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Summary. A fundamental relation between energy loss in electromagnetic engineering models leads to the definition of canonical stochastic fields. This canonical stochastic electromagnetic field model, which has been established in previous work (see [2–4]), is here used to construct stochastic processes weakly equivalent to the induced processes according to the canonical field model. Such processes can be used to test electronic systems, in particular communication links which are sensitive to specific correlation distances in induced noise.

1 Introduction

Because the environment in which an electronic system has to operate is not deterministically known, stochastic fields play an important role in electronic system testing. In this contribution, we elaborate on a fundamental relation between "energy loss" and stochastics generalising the well-known concept of noise temperature in electronics. The fundamental observation is that if a system's model shows loss of electromagnetic energy into some environment, it is implied that this environment works as a stochastic source of electromagnetic energy on the system in question.

We shall first recall the definition of a canonical stochastic electromagnetic field having a good space-time covariance operator. Having available this "a priori" model for the stochastic fields in the environment we can try to compute directly those characteristics of the signals which are decisive for our purpose. An example of this is the auto-covariance function of induced noise sources, which gives essential information on the internal structure this noise. In this contribution, we show a rather simple way to compute this auto-covariance function for a large class of problems. We also show how to generate noise realisations, functions of time, and compute the appropriate statistics from simulation results. This may be a practical strategy for essentially non-linear problems or when a direct method is not known.

2 Basic field theory

The energy emission operator of time-domain electromagnetic field theory is given by $C(J) = E(J) - \overline{E(\overline{J})}$ ($\overline{X}$ is the time reversal of X). This is the electric field propagator anti-symmetrised with respect to time reversal, i.e., $\overline{C(J)} = -C(\overline{J})$. The following integral relation justifies the name.

$$\int_{D\times\{t_1\}} (\mu_0 H^a \cdot H^b + \varepsilon_0 E^a \cdot E^b) = -\int_{D\times(t_0,t_1)} C(J^b)\cdot J^a \tag{1}$$

Here (t_0,t_1) is a time interval and the current distributions J^a and J^b have their spatial support in D and vanish outside the given time interval.

It has been shown in [4] that this energy emission operator is also the covariance operator, C_E, of a stochastic field defined by a probability measure on the space of initial values, $\mathscr{S}'(\mathbb{R}^3 \times \{t_0\})$, ($\mathscr{S}'$, here, denotes the vector-valued tempered distributions), i.e., by a stochastic distribution $\psi_0 = (e_0, h_0)$, such that

$$\forall f,g \in \mathscr{S} \quad \mathbb{E}(\langle \psi_0, f\rangle\langle\psi_0, g\rangle) = \sigma^2 \langle f,g\rangle_{\mathscr{H}}$$

where $\mathscr{S}$ is Schwartz' space of infinitely smooth test fields, $\mathscr{H}$ is the direct sum Hilbert space with the inner product given by the LHS of (1) and $\mathscr{S} \subset \mathscr{H} \subset \mathscr{S}'$ are dense inclusions (see [1]). We obtain for any two distributions J^a and J^b ,

$$\int_{D\times(t_0,t_1)} C_{E_0}(J^a)\cdot J^b = \sigma^2 \int_{D\times(t_0,t_1)} C(J^a)\cdot J^b \tag{2}$$

where E_0 is the electric field corresponding to the stochastic distribution ψ_0 on $D \times \{t_0\}$ and σ^2 a variance parameter of this stochastic initial value distribution.

3 Observables on stochastic fields

We now concentrate on Thévenin sources representing the action of electromagnetic fields, in some environment, on an electronic system placed in it. Thévenin sources are "observables" defined through distributions on electromagnetic fields. For example, $\mathscr{V} = \langle J_P, E\rangle = \int_P E$, where P is a curve defining an electronic port and E is the total electric field in the port region. If the electric field is a stochastic field, the given formula defines the Thévenin source as a generalised stochastic process.

An important characteristic is the auto-covariance of the Thévenin sources as function of time. Supposing that the average field is zero, we get,

$$\begin{aligned}\mathbb{E}(V(t_1)V(t_2)) &= \mathbb{E}(\langle J_{P,t_1}, E\rangle\langle J_{P;t_2}, E\rangle) \\ &= \langle J_{P,t_1}, C_E(J_{P,t_2})\rangle \end{aligned} \tag{3}$$

where C_E is the covariance operator of the stochastic field E. Thévenin sources have alternative integral representations in terms of a time reversed current distribution, $\overline{j(t)}$, on the conductors of the electronic system and only the incident part of the total electric field. The current distribution, $j(t)$, appearing in such a representation is the one appearing by applying a Dirac current source to the electronic port considered. This current distribution corresponds to the scattering of an elementary dipole field by the conductors of the system. The electric field, $e = C_E(j)$, appearing in (3), is the opposite of the trace of the elementary dipole field on the conductors. This simplifies (3) to $\mathbb{E}(V(t_1)V(t_2)) = \langle j(t_1), e(t_2)\rangle$, which is more easily computed than the general expression.

4 Statistically equivalent stochastic processes and the radiation resistance

The analysis presented thus-far, allows for estimating a priori covariances of interference sources to be applied to system models accounting for uncertain environments in a way consistent with the mechanisms of electromagnetic energy loss to this environment. In order to actually test a system by simulations using the stochastic environment model, we may want to generate time functions for the noise sources and compute statistics on the essential properties of the system. The question arises, then, whether the generated time functions are representative for those one will obtain in the actual situation. However, such a question has no answer, the only thing we can find out is whether a set of generated time functions is statistically equivalent to the actual processes we try to model. That means if we create sufficiently many functions with our algorithm, we want that the statistics we are interested in converge to the statistics we would obtain in the situation we model. In our case, we want the sample functions we generate to have an autocorrelation function with a good convergence to the autocorrelation function obtained in the actual situation.

We will obtain the desired result in two steps. The first step is to show how we can compute the spectral representation of the autocovariance function of an observable on a canonical stochastic field. The second step is to apply a standard Fourier integral representation trick to generate a stochastic process which has this auto covariance function.

The autocovariance of the observable Thévenin source V is given by

$$\begin{aligned} C_V(s,t) &= \mathbb{E}[\int_{\omega\in\mathbb{R}} V_\omega(s) \int_{\nu\in\mathbb{R}} V_\nu(t)] \\ &= \int_{\omega,\nu\in\mathbb{R}^2} \langle j_\omega(s), C_E j_\nu(t)\rangle \\ &= \int_{\omega,\nu\in\mathbb{R}^2} \langle \widehat{j_\omega} \exp(j\omega s), C_E \widehat{j_\nu} \exp(j\omega t)\rangle \end{aligned}$$

where $\widehat{j_\omega}$ is the spectral component of the current distribution defining the observable.

Using the stationarity of the processes, i.e. invariance under time translations, we can reduce the double integral over the frequencies to a single one and we get

$$\begin{aligned} \forall t\in\mathbb{R} \quad C_V(t,t+\tau) &= \int_{\omega\in\mathbb{R}} \langle j_\omega, E(j_\omega)\rangle \exp(j\omega\tau) \\ &= \int_{\omega\in\mathbb{R}} R_j(\omega)\exp(j\omega\tau) \end{aligned}$$

and $R_j(\omega)$ is the frequency domain radiation resistance of the current distribution j_ω.

For the second step, we use a well-known relation

$$\mathbb{E}[f(t)f(s)] = \int_{\omega\in\mathbb{R}} \mathrm{var}(\widehat{f}(\omega))\exp(j\omega(s-t)$$

valid for spectral amplitudes $\widehat{f}(\omega)$ and $\widehat{f}(\nu)$ statistically independent if $\nu \neq \pm\omega$, and, in addition satisfying $\mathbb{E}[\mathrm{Re}(\widehat{f}(\omega))\mathrm{Im}(\widehat{f}(\omega))] = 0$ and $\mathbb{E}[\mathrm{Re}(\widehat{f}(\omega))^2] = \mathbb{E}[\mathrm{Im}(\widehat{f}(\omega))^2]$. This result implies that a stochastic process which has spectral amplitudes satsifying the said constraints and have a variance equal to the frequency domain radiation resistance has the correct autocovariance function.

5 Conclusion

We obtain explicitly computable time functions which are realisations of a stochastic process statistically equivalent (with respect to the autocorrelation function) to an observable on a canonical stochastic field. This process includes the geometrical properties of the system, by means of the traversal times and the evaluation of the defining current distributions on the material configuration, but it also accounts for resonances in the configuration and between the configuration and the environment through the environment's Green function which defines the space-time covariance operator of the canonical stochastic field.

References

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